On the spatial variation of resistance to flow in upland channel networks

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[1] Detailed field measurements of channel properties and flow characteristics collected in the Ashley and Cropp catchments (New Zealand) are used to investigate the spatial variation of resistance to flow across upland channel networks. The application of the Darcy-Weisbach equation and semilogarithmic flow resistance relationships reveals that mean flow velocities calculated from local measurements of bed material particle size, hydraulic depth, and channel bed slope may be inaccurate. The Manning-Gauckler-Strickler equation with resistance coefficient independent of bed material particle size is found to be relatively more reliable but not sufficiently general to reproduce the spatial variation of resistance to flow across a complex channel network. A new methodology is developed by combining a hydraulic equation of the Manning-Gauckler-Strickler type, a flow discharge-upstream drainage area relationship, and geomorphological fluvial relationships for mean flow velocity, Gauckler-Strickler resistance coefficient, hydraulic depth, and friction slope. This methodology is found to improve the reproduction of the spatial variation of mean flow velocity across the Ashley catchment and appears of general applicability for the parameterization of resistance to flow in distributed catchment models. *INDEX TERMS:* 1824 Hydrology: Geomorphology (1625); 1848 Hydrology: Networks; 1860 Hydrology: Runoff and streamflow; *KEYWORDS:* resistance to flow, mountain streams, distributed catchment models parameterization

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1. Introduction

[2] Diffusion wave formulations such as those developed by *Hayami* [1951], *Cunge* [1969], *Ponce and Yevjevich* [1978], and *Ponce* [1986] are commonly recognized to be adequate mathematical descriptors of surface flow propagation in natural drainage systems [e.g., *Kellerhals*, 1970; *Beven and Wood*, 1993]. These formulations can capture the processes of advection and diffusion which normally determine the concentration of surface runoff and allow the control of stability, consistency (as referred to by *Ponce* [1986]), and diffusivity of numerical solutions [*Orlandini and Rosso*, 1996]. However, the effective computational capabilities of these formulations are qualified by the possibility to incorporate representative parameterizations of the drainage system. On the basis of the pioneering works of *O'Callaghan and Mark* [1984] and *Band* [1986], many procedures have been developed to extract the relevant physiographic features of a drainage system from digital elevation model (DEM) data [e.g., *Howard*, 1994; *Tarboton*, 1997]. These procedures are especially important to determine drainage directions, contributing areas, and slopes at all the sites in a catchment. As shown by *Howard* [1990] and *Orlandini and Rosso* [1998], geomorphological fluvial relationships introduced by *Kennedy* [1895], *Lacey* [1929], *Blench* [1951], *Leopold and Maddock* [1953], and *Leopold and Miller* [1956] for stable canals, rivers, and ephemeral streams may be employed to describe the hydraulic geometry of a fluvial system and also, heuristically, of upland channels and hillslope rills [e.g., *Osterkamp and Hedman*, 1977; *Parsons et al.*, 1994]. Resistance to flow is commonly described by selecting or calibrating resistance coefficients for equations of the Darcy-Weisbach (DW) or Manning-Gauckler-Strickler (MGS) type [e.g., *Barnes*, 1967; *Bathurst*, 1986].

[3] In this study, attention is focused on the description of resistance to flow in upland channel networks. The problem of resistance to flow concerns the prediction of the mean velocity of flow in terms of those channel properties and flow characteristics which act as a resistance or an energy loss to the flow. The variation of resistance to flow across a channel network produces characteristic variations in mean flow velocity along the fluvial system (downstream variations) and at given sites (at a station variations) that have important implications for flood routing, sediment and pollutant transport prediction, aquatic habitat management and other such concerns. At a station variations refer to the way in which mean flow velocity varies over time at a site, with changing discharge of different frequencies. Downstream variations refer to the way in which mean flow velocity varies in space along and between river channels (of a given fluvial system), with changing discharges of given frequency. At a station and downstream variations of mean flow velocity in natural channels have been described in the past using hydraulic flow resistance relationships

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based on the principles of fluid mechanics or using geomorphological fluvial relationships based on the concept of hydraulic geometry introduced by *Leopold and Maddock* [1953]. Hydraulic flow resistance relationships are recognized as being able to describe flows in regular channels (especially artificial channels) with good accuracy, but they can hardly ensure the same reliability for the description of flows in complex fluvial systems. Geomorphological fluvial relationships are recognized as being able to capture the essential features of at a station and downstream variations of mean flow velocity in complex fluvial systems, but they may oversimplify to some extent the general problem of resistance to flow as they do not establish comprehensive relations between mean flow velocity and relevant flow characteristics.

[4] Both hydraulic and geomorphological relationships are employed in this paper to assess the possibility of describing resistance to flow in upland channel networks. In particular, the possibility to reproduce the spatial variation of mean flow velocity across a complex channel network is evaluated using the extensive measurements of channel properties and flow characteristics collected during quasi-steady flow conditions across the Ashley and Cropp catchments (New Zealand). Firstly, the capabilities of hydraulic equations of the DW and MGS type to reproduce mean flow velocities from local measurements of channel properties and flow characteristics are evaluated using the Ashley catchment data, where numerous measurements of bed material particle size, hydraulic depth, and channel bed slope are available (section 3). Secondly, geomorphological fluvial relationships are used to reproduce the spatial variations of mean flow velocity in the two study catchments as explicit functions of the upstream drainage area (section 4). Thirdly, a hydraulic equation of the MGS type is combined with geomorphological fluvial relationships for mean flow velocity, Gauckler-Strickler (GS) resistance coefficient, hydraulic depth, and channel bed slope to provide a new formulation for the description of resistance to flow in complex channel networks (section 5). It is noted here that the field measurements used in this work are collected in the two study catchments during quasi-steady flow conditions and thus they depict only the spatial variations of channel properties and flow characteristics (in each of the two study catchments). Field measurements on temporal variations of flow characteristics are not available and thus these variations are not investigated in this work. Short comments are reported in section 5.3 to indicate how the formulation developed and tested in this work can be extended to describe flows that may occur in upland channel networks during flooding conditions.

2. Study Catchments

[5] The study presented in this paper is conducted using field data that were collected in the Ashley and Cropp catchments, located in the South Island of New Zealand. These data were made available through the data notes published by *McKerchar et al.* [1998] and *Henderson et al.* [1999]. The data include extensive measurements of channel properties and flow characteristics that were made over intervals of days (5 and 1, for the Ashley and Cropp catchment, respectively), when the river flow was reasonably steady. Hence they can be used to test concepts and assumptions on the spatial variations of channel properties and flow characteristics, but they do not allow analyses of the temporal variations of flow characteristics. Channel properties and flow characteristics were monitored, at least in part, in 336 and 65 sites across the Ashley and Cropp catchment, respectively. However, to allow the maximum simplicity and comparability of the methodologies examined or developed in the present work, only those sites where all the relevant measurements are available (or calculable) are considered. This requirement is met by 198 sites across the Ashley catchment and 46 sites across the Cropp catchment. Details on data acquisition, measurements techniques, checks applied, and data format are given by *McKerchar et al.* [1998] and *Henderson et al.* [1999], respectively. Part of the hydrologic characteristics of the Ashley and Cropp catchments which are published in these data notes are also reported below.

[6] The areas of the Ashley and Cropp catchments are about 121 and 13 km^2, respectively. Terrains of these two catchments are mountainous. Elevation ranges above sea level are about 500–1800 and 860–2000 m. The catchments are in a "near natural" condition with no human settlements. Vegetation is mainly native grassland or pasture for occasional grazing in the lower parts of the catchments. Native beech forest is present in the upper part of the Ashley catchment. There is erosion throughout the catchments, with gullying at the stream heads and extensive areas of sheet and scree erosion on hillsides. Stream channels are alluvial. The climate is temperate with precipitation throughout the year. The Ashley catchment drains to the South, and there is a rainfall gradient from about 900 mm yr^{-1} along the eastern boundary to about 1800 mm yr^{-1} along the western boundary. The Cropp catchment drains to the East, and mean annual rainfall is about 11,000 mm yr^{-1}. An annual potential evaporation of about 500 mm yr^{-1} is typical of the region. Mean annual flow discharge at the catchment outlets are 4.0 and 4.8 m^3 s^{-1} for the Ashley and Cropp catchment, respectively. Mean discharge over the 5-day measuring interval in the Ashley catchment, between February 8–12, 1994, was 1.6 m^3 s^{-1}, and over the 20-year recording interval this discharge was exceeded for 74% of the time. Mean discharge over the 1-day measuring interval in the Cropp catchment, on February 26, 1996, was 2.0 m^3 s^{-1}, and over the 17-year recording interval this discharge was exceeded for 62% of the time. Recent hydrological studies on the Ashley and Cropp catchments include those published by *Ibbitt* [1997] and *Ibbitt et al.* [1999].

3. Hydraulic Flow Resistance Relationships

[7] The capabilities of formulations that have some physical basis involving the principles of hydraulics to describe the spatial variation of resistance to flow across upland channel networks are evaluated in this section. Cross sections of channels are assumed to be wide (with respect to mean flow depths). This approximation can be expressed as either $R \simeq Y_m$ or $P \simeq W$, where $R = \Omega/P$ is the hydraulic radius, Ω is the cross-sectional flow area, P is the wetted perimeter, $Y_m = \Omega/W$ is the hydraulic depth (mean flow depth), and W is the water-surface width. The wide channel approximation appears reasonable for the cross sections considered in this work, which display average values of the aspect ratio W/Y_m in the order of 15, and also for most of

the cross sections in natural drainage systems [e.g., *Orlandini and Rosso*, 1998]. Two flow resistance relationships commonly used in channel flow routing (in the dynamic, diffusion, and kinematic wave approximations) are the DW equation

$$U = \left(\frac{8gY_mS_f}{f}\right)^{1/2}, \tag{1}$$

where U is the mean flow velocity, g is the acceleration due to gravity, S_f is the friction slope, and f is the DW resistance coefficient, and the MGS equation

$$U = k_S Y_m^{2/3} S_f^{1/2}, \tag{2}$$

where k_S is the GS resistance coefficient ($k_S = 1/n$, n being the Manning resistance coefficient). Under uniform flow conditions the local energy balance equation $S_f = S_o$ can be incorporated into (1) and (2). Both the DW and MGS equations account for resistance with a single coefficient, f and k_S (or n), respectively, and the central problem of channel flow resistance is the evaluation of these resistance coefficients. Combining (1) and (2) one can obtain the equation

$$k_S = (8g/f)^{1/2} Y_m^{-1/6}, \tag{3}$$

which provides the linkage between the DW and MGS formulations.

[8] The DW equation (1) and semilogarithmic relationships for the evaluation of the resistance coefficient f are commonly recognized as providing a sound basis for understanding the flow resistance variations over a wide range of conditions [e.g., *American Society of Civil Engineers*, 1963]. Three semilogarithmic (base 10) equations are selected among those published in the literature. For relative submergences $Y_m/D_{84} > 10$, D_{84} being the bed material particle size for which 84% of the material is finer, and channel bed slope $S_o < 0.040$, the relationship

$$\left(\frac{8}{f}\right)^{1/2} = 5.75 \log\left(\frac{3.51Y_m}{D_{84}}\right) \tag{4}$$

has been proposed by *Hey* [1979] and *Bathurst* [1982] for gravel bed rivers. For relative submergences $Y_m/D_{84} \leq 10$ and $S_o = 0.004$–0.040, the relationship

$$\left(\frac{8}{f}\right)^{1/2} = 5.62 \log\left(\frac{Y_m}{D_{84}}\right) + 4 \tag{5}$$

has been proposed by *Graf et al.* [1983] and *Bathurst* [1985] for boulder-bed rivers. For relative submergences $Y_m/D_{90} \leq 10$, D_{90} being the bed material particle size for which 90% of the material is finer, and $S_o = 0.040$–0.200, the relationship

$$\left(\frac{8}{f}\right)^{1/2} = 5.75\left[1 - \exp\left(-\frac{0.05Y_m}{D_{90}S_o^{1/2}}\right)\right]^{1/2} \log\left(\frac{8.2Y_m}{D_{90}}\right) \tag{6}$$

has been proposed by *Smart and Jaeggi* [1983] for steep pool-fall streams. Semilogarithmic relationships (4)–(6) are assumed in this work to constitute the best estimators of the values that the DW resistance coefficient f can take in mountain rivers, based on local measurements of channel and flow characteristics.

[9] The MGS equation (2) and empirical relationships for the evaluation of the resistance coefficient k_S probably constitute the most popular formulation for the description of resistance to flow in natural channels. *Strickler* [1923] and *Keulegan* [1938] proposed estimating k_S based on the bed material particle size as expressed by $k_S = 25.6\ D_{50}^{-1/6}$, D_{50} being the bed material particle size for which 50% of the material is finer, and $k_S = 28.6\ D_{90}^{-1/6}$, respectively. These relationships are recognized as providing reasonable estimates of k_S in low-gradient streams with bed material of gravel size or smaller, but are often found to overestimate k_S in steep mountain rivers [*Jarrett*, 1990]. This tendency can be confirmed using the data collected in the Ashely catchment. *Limerinos* [1970] related the GS resistance coefficient to hydraulic depth and bed material particle size as expressed by $k_S = [1.16 + 2.0 \log(Y_m/D_{84})]/(0.113\ Y_m^{0.16})$. This relationship is substantially equivalent to the combination of (3) and (5) and thus it is implicitly considered in the analysis based on the DW equation (1) and semilogarithmic relationships (4)–(6). *Jarrett* [1984, 1990] suggested estimating the GS resistance coefficient in steep mountain streams with slope up to 0.09 and low relative submergences using the relationship

$$k_S = 3.125\, Y_m^{0.16} S_f^{-0.38}. \tag{7}$$

This relationship with $S_f = S_o$ is assumed in this work to constitute the best estimator of the values that the GS resistance coefficient k_S can take in mountain rivers, based on local measurements of channel and flow characteristics. Note that relationship (7) does not incorporate the bed material particle size D_x (x = 50, 84, 90) and this essentially makes the MGS equation (2) a fluvial relationship of the form $U = 3.125 Y_m^{0.83} S_o^{0.12}$.

[10] Relationships (4)–(6) and (7) are tested using the Ashley catchment data set, where numerous quantitative descriptions of channel substrate allow the estimation of bed material particle size D_x (x = 84, 90) used in (4)–(6). These descriptions were obtained at the monitored sites by assessing visually the percentage of substrate within the following classes: sand and silt (grain size less than 2 mm), fine gravel (2–16 mm), gravel (16–64 mm), small cobbles (64–132 mm), large cobbles (132–250 mm), boulders (grain size greater than 250 mm), and bedrock. At each site, a cumulative frequency curve is derived from the estimated percentages of granular material and the related values of D_x (x = 84, 90) are obtained by linear interpolation. These values are adjusted for all those sites where bedrock was observed by considering the percentage of bedrock and a roughness length of 50 mm for this bedrock. The predictive capabilities of the hydraulic relationships considered in this section are evaluated using three functions of the errors between measured (U) and calculated ($\hat{U}$) values of mean flow velocity across the channel networks: (1) the mean error (ME), (2) the mean absolute error (MAE), and (3) the root mean square error (RMSE). The

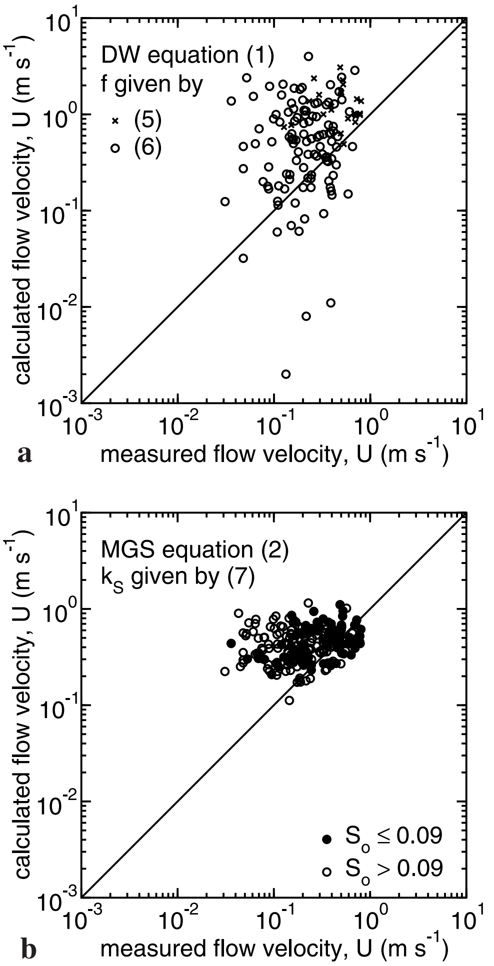

Figure 1. Comparison between measured and calculated flow velocities U in the Ashley catchment. (a) Predictive capabilities of the Darcy-Weisbach (DW) equation (1) and semilogarithmic flow resistance relationships (5)–(6) (Table 1). No data points are plotted for relationship (4) as the conditions for the application of this relationship are never met. (b) Predictive capabilities of the Manning-Gauckler-Strickler (MGS) equation (2) and relationship (7) (Table 1).

ME is defined as ME = $\Sigma_{i=1}^{N}(\hat{U}_i - U_i)/N$, N being the number of data points, and expresses the bias between measured and calculated flow velocities. The MAE is defined as MAE = $\Sigma_{i=1}^{N} |\hat{U}_i - U_i|/N$ and expresses the mean absolute deviation between measured and calculated flow velocities. The RMSE is defined as RMSE = $[\Sigma_{i=1}^{N}(\hat{U}_i - U_i)^2/N]^{1/2}$, and expresses a deviation between measured and calculated flow velocities as well as the MAE, but it emphasizes the large differences between measured and calculated values that may occur in a channel network.

[11] The combined application of the DW equation (1) (with $S_f = S_o$) and relationships (4)–(6) to the Ashley catchment data produces the results shown in Figure 1a. No data points are plotted for relationship (4) as the conditions for the application of this relationship are never met. Relationships (5)–(6) provide inaccurate reproductions of mean flow velocity across the catchment. The ME, MAE, and RMSE between measurements and calculations are 0.54, 0.59, and 0.87 m s^{-1}, respectively (Table 1). More reliable reproductions of mean flow velocity across the Ashley catchment are obtained from the combined use of the MGS equation (2) (with $S_f = S_o$) and relationship (7) (filled circles in Figure 1b). The ME, MAE, and RMSE between measurements and calculations are 0.13, 0.22, and 0.27 m s^{-1}, respectively (Table 1). The performance of relationship (7) under all the possible circumstances observed across the Ashley catchment ($S_o \leq 0.09$ and $S_o > 0.09$) is evaluated to test the capabilities of this relationship in catchment-wide applications (filled and empty circles in Figure 1b). The ME, MAE, and RMSE between measurements and calculations are 0.20, 0.24, and 0.30 m s^{-1}, respectively (Table 1). As shown in Figure 1b, the MGS equation (2) and relationship (7) provide reasonable estimates of the high flow velocities observed in the Ashley catchment, but overestimate the low flow velocities that generally occur in the upper part of the catchment, leading to a poor reproduction of the spatial variation of mean flow velocity across the catchment.

[12] The numerical experiments conducted in this section reveal that the DW equation (1) and semilogarithmic flow resistance relationships (5)–(6) may provide inaccurate predictions of mean flow velocity in upland channel networks. The formulation based on the MGS equation (2) and relationship (7) is found to be relatively more reliable but not sufficiently general to reproduce the spatial variation of mean flow velocity across a complex channel network. These results suggest that (1) bed material particle size D_x (x = 84, 90) is not a reliable indicator of resistance to flow in mountain streams where the size of bed material particles is comparable with the hydraulic depth of flow, and (2) geomorphological fluvial relationships may constitute a valid tool for providing flexible descriptions of resistance to flow in complex channel networks. Geomorphological fluvial relationships are employed to provide explicit descriptions of the spatial variation of mean flow velocity in section 4,

Table 1. Error Functions Between Measured and Calculated Flow Velocities

Mathematical Formulation	ME, m s^{-1}	MAE, m s^{-1}	RMSE, m s^{-1}
Ashley Catchment			
DW equation (1) and semilogarithmic relationships (5)–(6) (Figure 1a)	0.54	0.59	0.87
MGS equation (2) and relationship (7) ($S_o \leq 0.09$, Figure 1b)	0.13	0.22	0.27
MGS equation (2) and relationship (7) ($S_o \leq 0.09$ and $S_o > 0.09$, Figure 1b)	0.20	0.24	0.30
Fluvial relationship (10) (Table 2 and Figure 3c)	−0.03	0.08	0.11
MGS equation (2) and relationships (13)–(14) (Table 2 and Figures 5c and 5e)	−0.01	0.11	0.15
Cropp Catchment			
Fluvial relationship (10) (Table 2 and Figure 3d)	−0.03	0.10	0.13
MGS equation (2) and relationships (13)–(14) (Table 2 and Figures 5d and 5f)	−0.03	0.11	0.14

and are incorporated into a hydraulic equation of the MGS type to provide a more detailed description of resistance to flow in upland channel networks in section 5.

4. Geomorphological Fluvial Relationships

[13] Fluvial geomorphological studies by *Kennedy* [1895], *Lacey* [1929], *Blench* [1951], and *Leopold and Maddock* [1953] have shown that superimposed on the great heterogeneity among any group of stream channels there are certain broad generalizations that tie river channels into continua, on which certain characteristics seem to apply equally to a wide variety of cases. Described under the term "hydraulic geometry," these generalizations indicate that in responding to loads of water and sediment imposed on them, rivers change their forms in discernible ways. At a station and downstream variations of water-surface width, mean flow depth, mean flow velocity, suspended sediment load, channel bed slope, and Manning resistance coefficient are expressed as simple power function relationships of flow discharge, with different frequencies at a given site or with given frequency at different sites, respectively. The concept of hydraulic geometry is employed in this section to provide explicit descriptions of the spatial variation of mean flow velocity in complex channel networks. The downstream fluvial relationship for mean flow velocity U is given in the form

$$U = k'' Q^{m''}, \tag{8}$$

where Q is the flow discharge of a given frequency (probability of occurrence) at different points moving downstream in the channel network, k'' is a coefficient, and m'' is an exponent. The spatial variation of U is expressed through the variation of Q along the considered drainage system. Although this is not necessarily equivalent to discharge Q resulting from the same event at different points, downstream fluvial relationships are assumed in this study to be reasonable descriptors of what may occur in that situation. During quasi-steady flow conditions the spatial variation of flow discharge Q may be approximated with sufficient accuracy as a power function relationship of the upstream drainage area A, that is

$$Q = uA^{w}, \tag{9}$$

where u is a coefficient and w is an exponent, and thus by combining (8) and (9) one can obtain that the spatial variation of U can be expressed by a simple power function relationship of A, that is

$$U = k''' A^{m'''}, \tag{10}$$

where $k = u^{m''} k''$ and $m''' = w\, m''$.

[14] Parameters u, w, k''', and m''' in (9)–(10) are estimated for the Ashley and Cropp catchments by performing ordinary least squares (OLS) regressions on logarithmically transformed values of Q, U, and A. The coefficient of determination $R^2 = 1 - \Sigma_{i=1}^{N}(y_i - \hat{y}_i)^2/\Sigma_{i=1}^{N}(y_i - \bar{y})^2$, where N is the number of data points, y_i $(i = 1,\cdots, N)$ is the logarithm of the ith observed value for the considered flow characteristic, $\hat{y}_i$ $(i = 1,\cdots, N)$ is the fitted (calculated) value

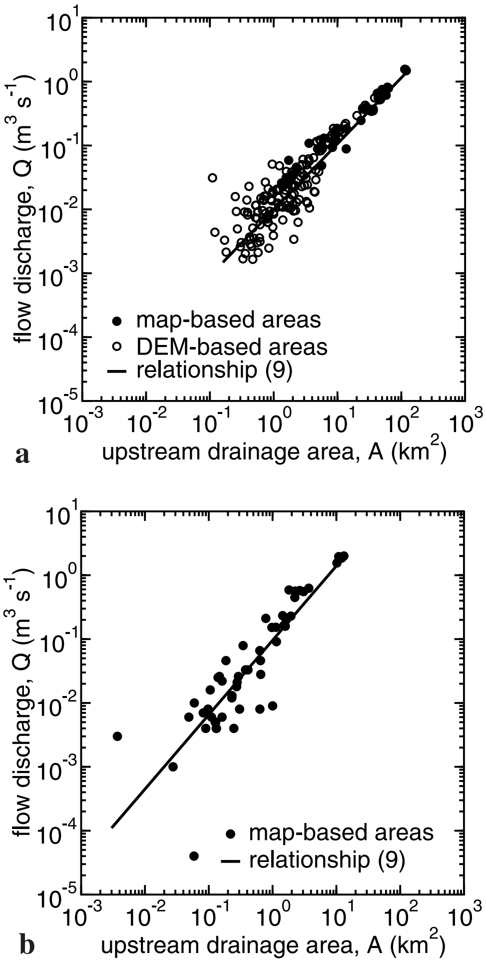

Figure 2. Spatial variations of flow discharge Q with upstream drainage area A observed and reproduced (relationship (9)) in the (a) Ashley and (b) Cropp catchments (Table 2).

of y_i, and $\bar{y}$ is the sample (arithmetic) mean of $(y_1,\cdots, y_N)$, is calculated to evaluate the overall goodness of fit of the estimated relationships with respect to the original data points [see, e.g., *Kottegoda and Rosso*, 1997, p. 371]. Upstream drainage areas in the Ashley catchment are measured directly from a topographic map, for several sites, and are calculated by mapping the locations reported on a channel network map and on a DEM-based channel network for many other sites. Map-based areas can be found in the data set described by *McKerchar et al.* [1998], whereas DEM-based areas are calculated in this work by using the algorithm described by *Orlandini and Rosso* [1998]. Using these data, the spatial variation of observed flow discharge Q with upstream drainage area A is approximated by the relationship Q = 0.013 $A^{0.978}$ with R^2 = 0.87 (Figure 2a and Table 2). The variation of mean flow velocity U with the upstream drainage area A is approximated by U = 0.149 $A^{0.310}$ with R^2 = 0.48 (Figure 3a and Table 2). The values of mean flow velocity predicted by this relationship are compared with measurements in Figure 3c. The related ME, MAE, and RMSE are −0.03, 0.08, and 0.11 m s^{-1}, respectively (Table 1). In the Cropp catchment upstream drainage areas are obtained from the map of the catchment

Table 2. Estimated Fluvial Relationships and Related Values of R^2[a]

Fluvial Relationship	Coefficient	Exponent	R^2
Ashley Catchment			
Flow discharge relationship (9) (Figure 2a)	0.013	0.978	0.87
Flow velocity relationship (10) (Figure 3a)	0.149	0.310	0.48
Hydraulic depth relationship (14) (Figure 4a)	0.099	0.212	0.41
Friction slope relationship (15) (Figure 4c)	0.178	−0.518	0.61
Water-surface width relationship (24) (Figure 4e)	0.867	0.456	0.74
GS resistance coefficient relationship (13) (Figure 5a)	1.649	0.427	0.49
Cropp Catchment			
Flow discharge relationship (9) (Figure 2b)	0.103	1.203	0.82
Flow velocity relationship (10) (Figure 3b)	0.246	0.199	0.18
Hydraulic depth relationship (14) (Figure 4a)	0.159	0.392	0.68
Friction slope relationship (15) (Figure 4d)	0.266	−0.304	0.49
Water-surface width relationship (24) (Figure 4f)	2.653	0.608	0.73
GS resistance coefficient relationship (13) (Figure 5b)	1.627	0.090	0.03

[a] Relationships $u = a''' c''' k'''$ and $w = b''' + f''' + m'''$ (obtained from $Q = WY_mU$, (9), (24), (14), and (10)) are essentially verified. For the Ashley catchment $u = 0.013$ m^3 s^{-1} km^{-2w}, $a''' c''' k''' = 0.013$ m^3 s^{-1} km^{-2w}, $w = 0.978$, and $b''' + f''' + m''' = 0.978$. For the Cropp catchment $u = 0.103$ m^3 s^{-1} km^{-2w}, $a''' c''' k''' = 0.104$ m^3 s^{-1} km^{-2w}, $w = 1.203$, and $b''' + f''' + m''' = 1.199$. The discrepancy between w and $b''' + f''' + m'''$ in the Cropp catchment is due to errors with which the relation $Q = WY_mU$ is verified by measurements that constitute the data set.

for all the sites as reported by *Henderson et al.* [1999]. Using these data, the spatial variation of observed flow discharge Q with upstream drainage area A is approximated by the relationship $Q = 0.103\ A^{1.203}$ with $R^2 = 0.82$ (Figure 2b and Table 2). The variation of mean flow velocity U with the upstream drainage area A is approximated by $U = 0.246\ A^{0.199}$ with $R^2 = 0.18$ (Figure 3b and Table 2). The values of mean flow velocity predicted by this

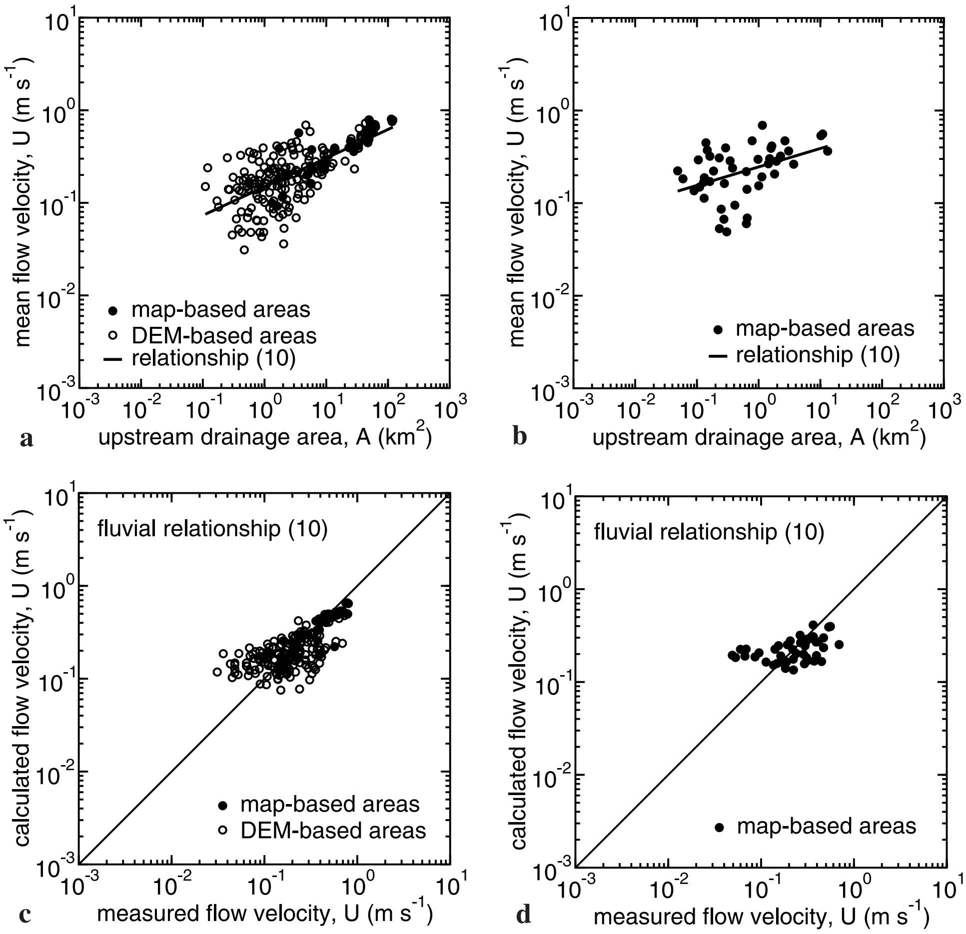

Figure 3. Spatial variations of mean flow velocity U with upstream drainage area A observed and reproduced (relationship (10)) in the (a) Ashley and (b) Cropp catchments (Table 2). Comparison between measured and calculated (relationship (10)) flow velocities U in the (c) Ashley and (d) Cropp catchments (Table 1).

relationship are compared with measurements in Figure 3d. The related ME, MAE, and RMSE are −0.03, 0.10, and 0.13 m s^{-1}, respectively (Table 1).

[15] It is noted here that relationship (9) is not meant to describe the downstream variation of peak discharge with upstream drainage area during flooding conditions, which is recognized to display exponent significantly less than 1, normally variable between 0.70 and 0.80 [*Leopold et al.*, 1964]. Relationship (9) is used in this paper to describe the spatial variation of flow discharge with upstream drainage area during quasi-steady flow conditions. Under these circumstances, the estimated relationships of the form (10) reproduce the spatial variations of mean flow velocity across the Ashley and Cropp catchments reasonably well (Figures 3c and 3d, respectively, and Table 1). However, these relationships do not allow a detailed description of resistance to flow, where flow velocity, flow geometry, and friction slope are functionally connected (e.g., equations (1) and (2)). A possible strategy to incorporate the flexibility of geomorphological relationships without losing the descriptive capabilities of hydraulic relationships may be to combine both these kinds of relationships into a single formulation. This strategy is developed and tested in section 5.

5. Combining Hydraulic and Geomorphological Relationships

[16] In this section, a hydraulic equation of the MGS type is combined with geomorphological relationships for mean flow velocity, GS resistance coefficient, hydraulic depth, and friction slope to provide a comprehensive description of resistance to flow in upland channel networks. As the available data sets depict the spatial variations of channel properties and flow characteristics during quasi-steady flow conditions, only downstream fluvial relationships are considered. The methodology for combining the MGS equation and geomorphological relationships is illustrated in section 5.1 and tested on the Ashley and Cropp catchment data in section 5.2. Short comments on the possibility to extend the proposed methodology for the description of flows occurring under flooding conditions are provided in section 5.3.

5.1. Methodological Aspects

[17] The MGS equation (2) can be solved for k_S so as to obtain

$$k_S = \frac{U}{Y_m^{2/3} S_f^{1/2}}. \tag{11}$$

Using the concept of hydraulic geometry proposed by *Leopold and Maddock* [1953], the downstream variation of k_S with flow discharge Q of given frequency can be expressed in the simple power function form

$$k_S = r'' Q^{y''}, \tag{12}$$

where r'' is a coefficient and y'' is an exponent. By combining relationships (12) and (9) one can find that, during quasi-steady flow conditions, the spatial variation of k_S with the upstream drainage area A can be expressed in the form

$$k_S = r''' A^{y'''}, \tag{13}$$

where $r = u^{y''} r''$ and $y = w\, y''$. Incorporating into (11) the fluvial relationships (13), (10), along with

$$Y_m = c''' A^{f'''}, \tag{14}$$

and

$$S_f = t''' A^{z'''}, \tag{15}$$

where c''' and t''' are coefficients, whereas f''' and z''' are exponents, one can obtain

$$r''' = \frac{k'''}{c'''^{2/3} t'''^{1/2}} \tag{16}$$

and

$$y''' = m''' - \frac{2}{3} f''' - \frac{1}{2} z'''. \tag{17}$$

Equations (16) and (17) provide the linkage between the MGS equation (2) and geomorphological fluvial relationships of the flow characteristics used in this equation. These equations allow one to identify a reliable spatial distribution (with A) of the resistance coefficient k_S used in the MGS equation (2), given the estimated spatial variations (with A) of U, Y_m, and S_f. The variations of U and Y_m, as described by (10) and (14), can be estimated directly from field measurements (data points (A, U) and (A, Y_m), respectively) by means of OLS regressions on logarithmically transformed variables. The variation of S_f, as described by (15), can be estimated by assuming that the average trend of the observed data points (A, S_o) is representative of that of the unknown points (A, S_f), and by performing an OLS regression on logarithmically transformed variables (log S_o (for log S_f) and log A). The hydrologic significance of this assumption is discussed as follows.

[18] The energy per unit weight of water at a given fluvial section of a channel network is given by the total head $H = z_b + Y \cos\beta + U^2/(2g)$, where z_b is the elevation of the channel bed, Y is the flow depth, β is the channel bed inclination angle, and $U^2/(2g)$ is the velocity head. In upland channel networks, the loss of energy connected to each water element flowing along a sufficiently long fluvial path 1–2 can be approximated as

$$H_1 - H_2 \simeq z_{b1} - z_{b2}, \tag{18}$$

where subscripts 1 and 2 refer to the upstream and downstream sections of the path 1–2, respectively. Under these circumstances, the differences $Y_1 - Y_2$ and $U_1^2/(2g) - U_2^2/(2g)$ are negligible with respect to the difference $z_{b1} - z_{b2}$, $|Y_1 - Y_2|$ and $|U_1^2/(2g) - U_2^2/(2\,g)|$ being in the order of tens of centimeters and $z_{b1} - z_{b2}$ being in the order of tens of meters. If $S_f = -dH/ds$ denotes the local (unknown) value of the friction slope at the spatial coordinate s along the fluvial path 1–2, the difference $H_1 - H_2$ in (18) (energy per unit weight of water dissipated in the flow by frictional forces between sections 1 and 2) can be expressed as

$$H_1 - H_2 = \int_{s_1}^{s_2} S_f\, ds. \tag{19}$$

In addition, if $S_o = -dz_b/ds$ denotes the local value of channel bed slope ($S_o = \sin\beta$) at the spatial coordinate s

along the fluvial path 1–2, the difference $z_{b1} - z_{b2}$ in (18) (energy per unit weight of water provided to the flow by gravitational forces between sections 1 and 2) can be expressed as

$$z_{b1} - z_{b2} = \int_{s_1}^{s_2} S_o \, ds. \tag{20}$$

Incorporating (19) and (20) into the energy balance (18) yields that along a sufficiently long fluvial path 1–2

$$\int_{s_1}^{s_2} (S_f - S_o) \, ds \simeq 0. \tag{21}$$

[19] Equation (21) can be expressed using the upstream drainage area A as integration variable, that is

$$\int_{A_1}^{A_2} (S_f - S_o)(dA/ds)^{-1} dA \simeq 0, \tag{22}$$

where A_1 and A_2 are the upstream drainage areas at sections 1 and 2, respectively. Approximating the variation of A along the generic fluvial path 1–2 by straight line, the term dA/ds in (22) can be represented by a positive constant value and (22) can be written in the form

$$\int_{A_1}^{A_2} (S_f - S_o) \, dA \simeq 0. \tag{23}$$

Although data points (A, S_o) collected across a channel network refer to different fluvial paths and the variations of A with s along each of these fluvial paths are not exactly linear, the use of the data points (A, S_o) as substitutes of the unknown data points (A, S_f) in the OLS estimation of relationship (15) may provide a useful approximation of the overall energy balance conditions expressed by (21) and (23) across an upland channel network. In this light, the nonlocal mapping between S_f and S_o under consideration may allow one to incorporate the essential effects of channel bed slope S_o and to balance these effects with those produced by energy dissipation processes, due to frictional forces along the wetted perimeter and to other more complex hydraulic phenomena. Dispersion of data points (A, S_o) around the estimated relationships (15) reflects the occurrence of deviations from the local energy balance $S_f = S_o$ that would occur under uniform flow conditions, but not necessarily from the nonlocal energy balance expressed by (21) and (23), which has more general validity. The energetic significance of (21) and (23) must be qualified by the negligibility of the differences in flow depth and velocity head with respect to the differences in channel bed elevation along the drainage system. The term ‘‘upland channel network’’ is used in the present study to emphasize this fact.

[20] One can note that the power function relationship (11) between k_S, U, Y_m, and S_f can be expressed as a linear relationship between the logarithms of these variables. This implies that, if no missing data affect the data set, the estimates of r''' and y''' given by relationships (16) and (17) can also be obtained directly from an OLS regression of logarithmically transformed values of k_S (given by (11) with $S_f = S_o$) on logarithmically transformed values of A. The estimation procedure based on (16) and (17) is less expeditious than the direct OLS regression of log k_S on log A, but offers the possibility to evaluate the role played by each factor connected to resistance to flow in the quantification of k_S. The spatial variation of mean flow velocity across an upland channel can be reproduced using the MGS equation (2) with k_S given by (13), (16), and (17), Y_m given by (14), and S_f locally set equal to S_o. The concomitant use of the overall set of points (A, S_o), for the estimation of the spatial variation of k_S with A, and the local values of S_o as substitute of S_f, for the prediction of flow velocity based on (2), may allow one to control the overall energy balance across a complex channel network while also retaining consideration of the effects that local channel bed slope S_o has on mean flow velocity U. The resulting formulation provides mean flow velocity U at any location across a channel network in terms of upstream drainage area A and channel bed slope S_o, which can be automatically derived from the DEM data of the catchment.

[21] One can also note that, by using relationships (9), (14) and a similar geomorphological relationship for water-surface width W, that is

$$W = a''' A^{b'''}, \tag{24}$$

where a''' is a coefficient and b''' is an exponent, mean flow velocity can be calculated as $U = Q/(W\,Y_m)$, and the use of the MGS resistance equation (2) appears to be not strictly required [*Kellerhals*, 1970]. If no missing data affect the data set, coefficients and exponents of relationships $Q = W\,Y_m U$, (9), (24), (14), and (10) are connected by relationships $u = a'''\,c'''\,k'''$ and $w = b''' + f''' + m'''$, and the predictions that one may obtain from these relationships are the same as those provided by relationship (10). The use of the MGS equation (2) in the methodology developed in this section is motivated by the need to provide a detailed description of resistance to flow across complex channel networks, which can be employed for the parameterization of diffusion wave formulations based on the theory developed by *Hayami* [1951]. This theory requires a functional relationship between flow velocity, flow geometry, and friction slope [e.g., *Orlandini and Rosso*, 1998]. This motivation will be further clarified in section 5.3.

5.2. Catchment Applications

[22] As shown in Figure 4a, in the Ashley catchment the observed data points (A, Y_m) are interpreted by the relationship $Y_m = 0.099\,A^{0.212}$ with $R^2 = 0.41$ (Table 2). The observed data points (A, S_o) are used to determine the relationship $S_f = 0.178\,A^{-0.518}$ with $R^2 = 0.61$ (Figure 4c and Table 2). The observed data points (A, W) are interpreted by the relationship $W = 0.867\,A^{0.456}$ with $R^2 = 0.74$ (Figure 4e and Table 2). From equations (16) and (17), the spatial variation of k_S given by $k_S = 1.649\,A^{0.427}$ is obtained (Table 2). The same relationship can be obtained directly from an OLS regression of the logarithms of k_S given by

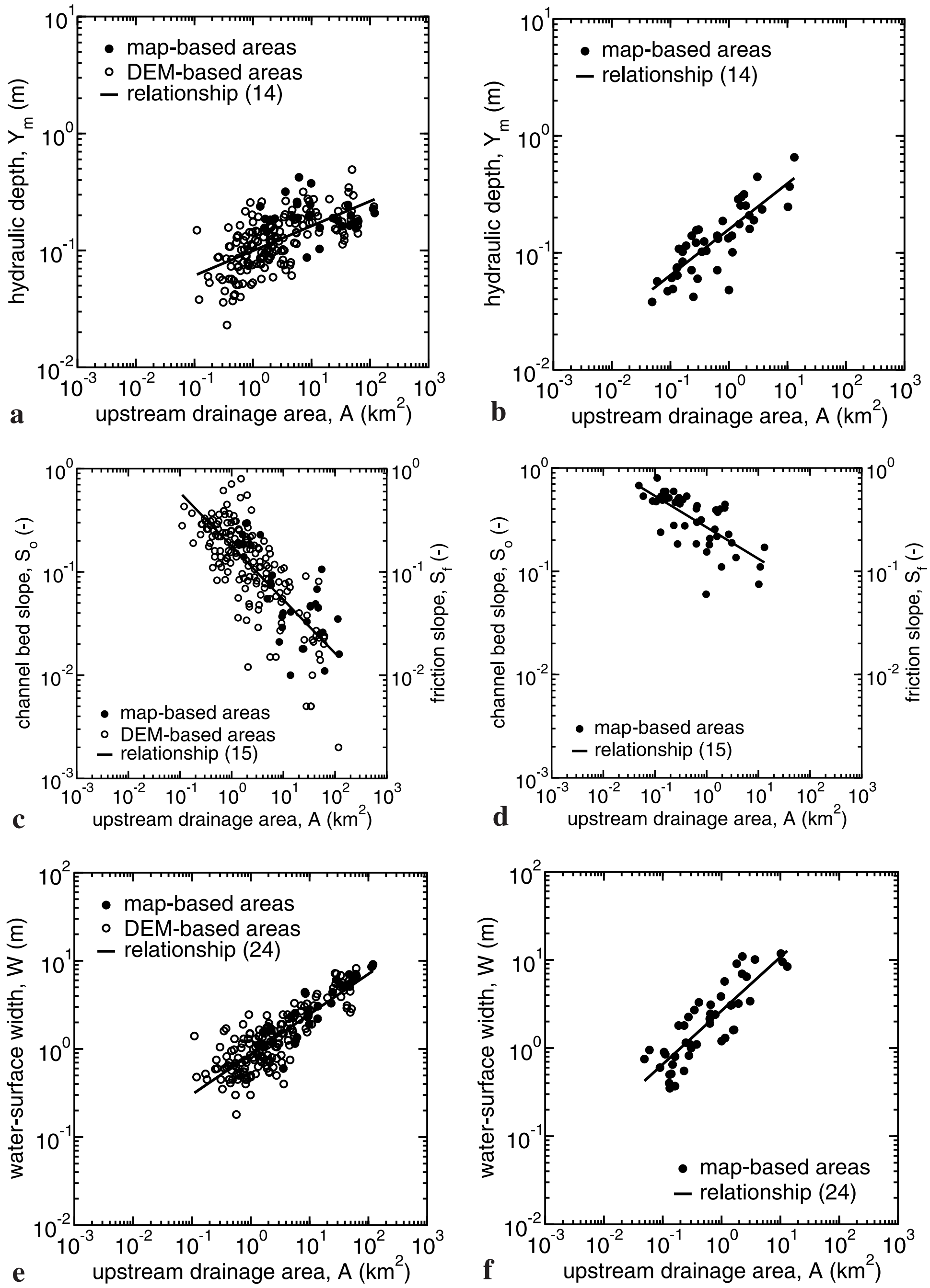

Figure 4. Spatial variations of hydraulic depth Y_m with upstream drainage area A observed and reproduced (relationship (14)) in the (a) Ashley and (b) Cropp catchments (Table 2). Observed spatial variations of channel bed slope S_o with upstream drainage area A and reproduced spatial variations of the friction slope S_f with upstream drainage area A (relationship (15)) in the (c) Ashley and (d) Cropp catchments (Table 2). Spatial variations of water-surface width W with upstream drainage area A observed and reproduced (relationship (24)) in the (e) Ashley and (f) Cropp catchments (Table 2).

(11) on the logarithms of A (Figure 5a). The related coefficient of determination is R^2 = 0.49 (Table 2). The observed values of k_S (obtained from (11) with $S_f = S_o$) range from 0.18 to 44.02 $m^{1/3}\ s^{-1}$ with an average of 4.36 $m^{1/3}\ s^{-1}$ (Table 3). The reproduced values of k_S range from 0.77 $m^{1/3}\ s^{-1}$ (for A = 0.11 km^2) to 11.22 $m^{1/3}\ s^{-1}$ (for A = 119.98 km^2) with an average of 3.33 $m^{1/3}\ s^{-1}$ (Table 3). Mean flow velocities calculated from the MGS equation (2) with k_S given by (13), Y_m given by (14), and S_f given by measurements of S_o are compared to measurements in Figure 5c. The ME, MAE, and RMSE between calculations and measurements are −0.01, 0.11, and 0.15 m s^{-1}, respectively (Table 1). The variation of the Froude number $Fr = U/(g\ Y_m)^{1/2}$ over the reproduced flow velocities is shown in Figure 5e, where circles of diameter proportional to Fr are plotted around the data points denoting the

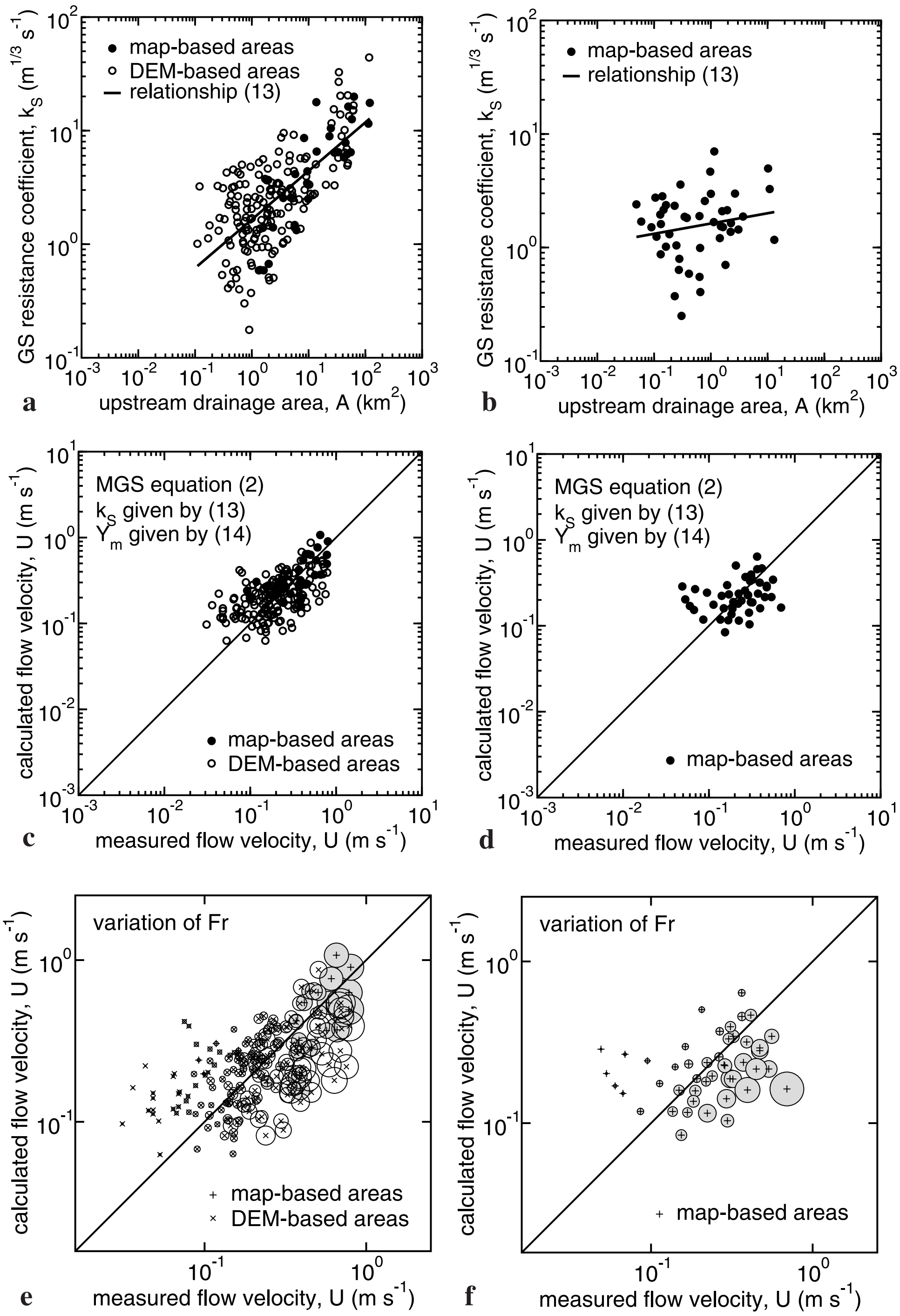

Figure 5. Spatial variations of the GS resistance coefficient k_S with upstream drainage area A estimated from local measurements as $k_S = U/(Y_m^{2/3}S_o^{1/2})$ and reproduced using relationship (13) in the (a) Ashley and (b) Cropp catchments (Table 2). Comparison between measured and calculated (MGS equation (2) with k_S given by (13) and Y_m given by (14)) flow velocities U in the (c) Ashley and (d) Cropp catchments (Table 1). Variations of the Froude number Fr (diameter of circles around markers) over the reproduced flow velocities (markers) in the (e) Ashley and (f) Cropp catchments. Froude number ranges are 0.027–0.698 in the Ashley catchment (Figure 5e) and 0.039–0.696 in the Cropp catchment (Figure 5f).

measured and calculated flow velocities. In the Ashley catchment Fr ranges from 0.027 to 0.698 with average 0.230.

[23] As shown in Figure 4b, in the Cropp catchment the observed data points (A, Y_m) are interpreted by the relationship $Y_m = 0.159\ A^{0.392}$ with $R^2 = 0.68$ (Table 2). The observed data points (A, S_o) are used to determine the relationship $S_f = 0.266\ A^{-0.304}$ with $R^2 = 0.49$ (Figure 4d and Table 2). The observed data points (A, W) are interpreted by the relationship $W = 2.653\ A^{0.608}$ with $R^2 = 0.73$ (Figure 4f and Table 2). From equations (16) and (17), the spatial variation of k_S given by $k_S = 1.627\ A^{0.090}$ is obtained

Table 3. Descriptive Statistics on Observed and Reproduced Values of the GS Resistance Coefficient

Statistic	Observed Value	Reproduced Value
	Ashley Catchment	
Minimum	0.18	0.77
Mean	4.36	3.33
Maximum	44.02	11.22
	Cropp Catchment	
Minimum	0.25	1.24
Mean	1.91	1.56
Maximum	7.02	2.05

(Table 2). The same relationship can be obtained directly from an OLS regression of the logarithms of k_S given by (11) on the logarithms of A (Figure 5b). The related coefficient of determination is R^2 = 0.03 (Table 2). The observed values of k_S (obtained from (11) with $S_f = S_o$) range from 0.25 to 7.02 $m^{1/3}$ s^{-1} with an average of 1.91 $m^{1/3}$ s^{-1} (Table 3). The reproduced values of k_S range from 1.24 $m^{1/3}$ s^{-1} (for A = 0.05 km^2) to 2.05 $m^{1/3}$ s^{-1} (for A = 13.14 km^2) with an average of 1.56 $m^{1/3}$ s^{-1} (Table 3). The weak variation of k_S with A in the Cropp catchment explains, at least in part, the low value of R^2 (0.03) obtained in this case. Mean flow velocities calculated from the MGS equation (2) with k_S given by (13), Y_m given by (14), and S_f given by measurements of S_o are compared to measurements in Figure 5d. The ME, MAE, and RMSE between calculations and measurements are −0.03, 0.11, and 0.14 m s^{-1}, respectively (Table 1). The variation of the Froude number $Fr = U/(g\ Y_m)^{1/2}$ over the reproduced flow velocities is shown in Figure 5f, where circles of diameter proportional to Fr are plotted around the data points denoting the measured and calculated flow velocities. In the Cropp catchment Fr ranges from 0.039 to 0.696 with average 0.232.

5.3. Possible Extensions

[24] The methodology developed in section 5.1 and tested in section 5.2 is designed to provide reliable parameterizations of the spatial variation in resistance to flow across upland channel networks. These parameterizations must be associated to the particular quasi-steady flow conditions under which parameters r''' and y''' are estimated. The description of the variation in resistance to flow in space (with upstream drainage area) and time (with flow discharge) requires three additional steps. Firstly, the exponents that express the at a station variations (with flow discharge) of mean flow velocity, GS resistance coefficient, and hydraulic depth must be estimated by considering flows of variable frequency. Secondly, a comprehensive description of resistance to flow may be derived by combining at a station and downstream fluvial relationships, and a hydraulic equation of the MGS type. Thirdly, the obtained parameterization may be incorporated into a diffusion waveformulation based on the theory developed by *Hayami* [1951]. A procedure similar to that outlined in this section is developed by *Orlandini and Rosso* [1998], where the at a station and downstream variations of water-surface width are used to describe the spatial and temporal variations of stream-channel-geometry in a complex channel network.

6. Discussion

[25] The analysis carried out in section 3 reveals that the spatial variation of mean flow velocity U in upland channel networks may be inaccurately reproduced through the iterative use of the DW equation (1) and detailed semilogarithmic relationships (5)–(6), even when accurate local measurements of bed material particle size D_x (x = 84, 90), hydraulic depth Y_m, and channel bed slope S_o are available (Figure 1a). Predicted velocities are affected by great bias (ME = 0.54 m s^{-1}, Table 1) and dispersion (MAE = 0.59 m s^{-1}, RMSE = 0.87 m s^{-1}, Table 1). The combined use of the MGS equation (2) and the *Jarrett*'s [1984] relationship (7) appears relatively more reliable, but it still produces considerable bias (ME = 0.13 m s^{-1}, Table 1) and dispersion (MAE = 0.22 m s^{-1}, RMSE = 0.27 s^{-1}, Table 1) in mean flow velocity predictions, especially when low flow velocities (which generally occur in the upper part of the channel network) are considered (Figure 1b). The poor ability of (2) and (7) to reproduce mean flow velocities in the upper part of the channel network is also indicated by the worsening in the performance of this formulation observed when also considering channels with slope $S_o > 0.09$ (ME = 0.20 m s^{-1}, MAE = 0.24 m s^{-1}, RMSE = 0.30 m s^{-1}, Table 1, Figure 1b). Nevertheless, the significant improvement obtained using the MGS equation (2) and relationship (7) (where the GS resistance coefficient k_S is not related to D_x (x = 84, 90)) in preference to the DW equation (1) and semilogarithmic relationships (5)–(6), suggest that, as currently used, local measurements of bed material particle size D_x (x = 84, 90) may not be reliable descriptors of the mechanisms that determine resistance to flow in upland channel networks. Mean flow velocities calculated in section 3 appear generally overestimated (ME > 0) with respect to measurements and this provides, at least in part, an explanation of the tendency of detailed distributed catchment models to produce flow hydrographs that are in advance (in time) of the corresponding observed hydrographs over the whole curves [*Orlandini and Rosso*, 1998].

[26] The numerical experiments conducted in section 3 suggest that, at least for mountain streams where the size of bed material is comparable with the depth of flow, considerable research efforts are required to provide reliable descriptions of resistance to flow based on local flow measurements. In the absence of satisfactory local formulations, the parameterization of resistance to flow across a complex channel network may rather be derived by imposing, although in an approximate manner, the overall balance between energy provided to the flows by gravitational forces and energy dissipated in the flows by frictional forces, and by employing geomorphological fluvial relationships to incorporate the essential variations of flow characteristics across the channel network. A preliminary step is carried out in section 4 to provide explicit descriptions of the spatial variations of mean flow velocity U in the Ashley and Cropp catchments during quasi-steady flow conditions. Under these circumstances, the spatial variations of flow discharge Q are satisfactorily reproduced in terms of upstream drainage area A by means of relationship (9) (Figure 2 and Table 2), and the spatial variations of mean

flow velocity U can be expressed directly in terms of upstream drainage area A by means of relationship (10) (Figure 3 and Table 2). The estimated relationships of the form (10) are found to reproduce reasonably well the observed flow velocities in the Ashley (ME = −0.03 m s^{-1}, MAE = 0.08 m s^{-1}, RMSE = 0.11 m s^{-1}) and Cropp (ME = −0.03 m s^{-1}, MAE = 0.10 m s^{-1}, RMSE = 0.13 m s^{-1}) catchments (Table 1). However, they appear not sufficiently detailed to serve as a basis for the parameterization of diffusion wave models derived from the theory developed by *Hayami* [1951]. This theory requires a functional relationship between flow velocity, flow geometry, and friction slope that may represent the essential features of resistance to flow across a fluvial system [e.g., *Orlandini and Rosso*, 1998].

[27] Geomorphological fluvial relationships are combined in section 5 with a hydraulic equation of the MGS type to provide a reasonably detailed and reliable description of the spatial variation of resistance to flow across a complex channel network. Relationships (10), (14), and (15) are estimated using the Ashley and Cropp catchment data to reproduce the essential features of the spatial variations (with upstream drainage area A) of mean flow velocity U, hydraulic depth Y_m, and friction slope S_f (Figures 3 and 4, Table 2). The overall balance between energy provided to the flows by gravitational forces and energy dissipated in the flows by frictional forces is ensured by using the spatial variation (with A) of channel bed slope S_o as substitute for the spatial variation (with A) of S_f (Figures 3c and 3d, for the Ashley and Cropp catchment, respectively, Table 2). The spatial variation (with A) of the GS resistance coefficient k_S can either be obtained from relationships (16) and (17) or from the direct OLS regression of logarithmically transformed values of k_S (given by (11) with $S_f = S_o$) on logarithmically transformed values of A (Figures 5a and 5b, for the Ashley and Cropp catchment, respectively, Table 2). With respect to the relationships considered in section 3, the MGS equation (2) with k_S given by (13), (16), and (17), Y_m given by (14), and S_f given by the observed values of S_o provides an improved reproduction of mean flow velocities U in the Ashley catchment (ME = −0.01, MAE = 0.11, and RMSE = 0.15 m s^{-1}, Table 1 and Figure 5c). With respect to the relationships considered in section 4, it is noted here that an important difference between the formulations developed in section 5 (relationships (2), (13), (16), (17), and (14)) and section 4 (relationship (10)) is connected to the use of local measurements of channel bed slope S_o for the prediction of mean flow velocity. This produces a reduction in the ME and an increase in MAE and RMSE, in the case of the Ashley catchment, and no significant changes in ME, MAE, and RMSE, in the case of the Cropp catchment (ME = −0.03, MAE = 0.11, and RMSE = 0.14 m s^{-1}, Table 1 and Figure 5d).

[28] The methodology developed and tested in section 5 must not be expected to provide accurate local predictions of mean flow velocity U at all the monitored sites of a channel network (Figure 5). The limits of the proposed methodology are expressed, at least in part, by plotting the variation of the Froude number Fr over the reproduced flow velocities (Figures 5e and 5f). One can note that the major deviations between measured and calculated flow velocities are generally connected to low (∼0.03) or high (∼0.70) values of Fr with respect to the average values (∼0.23). These circumstances are likely to be connected to the presence of pools in the channel networks and thus to the control of geology on the morphology of the river systems. The methodology developed in section 5 may incorporate the average effects that abrupt spatial variations of water-surface width W produce on resistance to flow (i.e., on the estimated spatial variations of k_S) but cannot reproduce accurately the effects of river morphology on local flow velocity predictions. However, the combined use of the reproduced spatial variations (with A) of k_S and Y_m, and of the observed values of S_o (for S_f) allow predictions of mean flow velocity U which are globally less biased and more accurate than those obtained from the iterative use of hydraulic flow resistance relationships (Figures 1 and 5 and Table 1). In addition, the methodology developed in section 5 expresses the spatial variations of resistance to flow using A and S_o as explicative variables and thus it can provide predictions of U, k_S, and Y_m at each point of a channel network, where DEM-based values of A and S_o are calculable [e.g., *Orlandini and Rosso*, 1998]. The proposed methodology is found to produce useful predictions of mean flow velocity U in catchments where the GS resistance coefficient k_S significantly varies with A, such as the Ashley catchment (Tables 2 and 3 and Figure 5a), and also in catchments where k_S does not vary significantly with A, such as the Cropp catchment (Tables 2 and 3 and Figure 5b), and thus appears of general applicability for the parameterization of distributed catchment models.

7. Conclusions

[29] The numerical experiments conducted in section 3 reveal that the iterative use of hydraulic flow resistance relationships based on local measurements of channel properties and flow characteristics may produce inaccurate spatial variations of mean flow velocity across a complex channel network (Figure 1). Predicted flow velocities appear affected by considerable positive bias and dispersion with respect to measurements (Table 1). The comparison between results obtained from the application of the DW equation (1) and the MGS equation (2) suggest that, as currently used, bed material particle size may not be a reliable indicator of resistance to flow in mountain streams (Figure 1 and Table 1). A reliable methodology for the description of the spatial variation of resistance to flow in upland channel networks is developed by combining a hydraulic equation of the MGS type and geomorphological fluvial relationships that express the variations with upstream drainage area of mean flow velocity, GS resistance coefficient, hydraulic depth, and friction slope (section 5). This methodology ensures, although in an approximate manner, the overall balance between energy provided to the flows by gravitational forces and energy dissipated in the flows by frictional forces, while also retaining consideration of the essential spatial variations of channel properties and flow characteristics within the catchment (Figures 3, 4, and 5 and Table 2).

[30] The methodology developed in section 5 is found to produce predictions of mean flow velocity that are significantly less biased and more accurate than those obtained

from the hydraulic relationships considered in section 3 (Table 1). These predictions are ultimately expressed in terms of upstream drainage area and channel bed slope, and thus they can be provided at each point of a channel network, where upstream drainage area and channel bed slope can be automatically estimated from DEM data. It is specified here that the spatial variations of mean flow velocity which one may obtain from the developed methodology must be associated to the particular quasi-steady flow conditions under which geomorphological fluvial relationships are estimated. Future work is required to evaluate experimentally the possibility to extend the developed formulation for incorporating the temporal variations (with flow discharge) of mean flow velocity, GS resistance coefficient, and hydraulic depth, so as to provide a comprehensive description of flow characteristics across an upland channel network during flooding conditions (section 5.3). Even at the present stage, the formulation developed in section 5 provides useful indications on how to parameterize the resistance to flow in distributed catchment models.

Notation

A upstream drainage area, km^2.
a''' coefficient of the spatial variation of water-surface width W with upstream drainage area A, m km$^{-2b'''}$.
b''' exponent of the spatial variation of water-surface width W with upstream drainage area A, dimensionless.
c''' coefficient of the spatial variation of hydraulic depth Y_m with upstream drainage area A, m km$^{-2f'''}$.
D_x bed material particle size for which x% (x = 50, 84, 90) of the material is finer, m.
f DW resistance coefficient, dimensionless.
f''' exponent of the spatial variation of hydraulic depth Y_m with upstream drainage area A, dimensionless.
Fr Froude nulmber, dimensionless.
g acceleration due to gravity, m s^{-2}.
H total head, m.
k'' coefficient of the spatial variation of mean flow velocity U with flow discharge Q, m$^{1-3\ m''}$ s$^{m''-1}$.
k''' coefficient of the spatial variation of mean flow velocity U with upstream drainage area A, m s^{-1} km$^{-2m'''}$.
k_S GS resistance coefficient ($k_S = 1/n$), m$^{1/3}$ s^{-1}.
m'' exponent of the spatial variation of mean flow velocity U with flow discharge Q, dimensionless.
m''' exponent of the spatial variation of mean flow velocity U with upstream drainage area A, dimensionless.
N number of data points.
n Manning resistance coefficient ($n = 1/k_S$), m$^{-1/3}$ s.
P wetted perimeter, m.
Q flow discharge, m^3 s^{-1}.
R hydraulic radius, m.
R^2 coefficient of determination, dimensionless.
r'' coefficient of the spatial variation of GS resistance coefficient k_S with flow discharge Q, m$^{1/3-3\ y''}$ s$^{y''-1}$.
r''' coefficient of the spatial variation of GS resistance coefficient k_S with upstream drainage area A, m$^{1/3}$ s^{-1} km$^{-2y'''}$.
S_o channel bed slope ($S_o = \sin\beta$), dimensionless.
S_f friction slope, dimensionless.
s spatial coordinate along a fluvial path, m.
t''' coefficient of the spatial variation of friction slope S_f with upstream drainage area A, km$^{-2z'''}$.
U mean flow velocity, m s^{-1}.
u coefficient of the spatial variation of flow discharge Q with upstream drainage area A, m^3 s^{-1} km^{-2w}.
W water-surface width, m.
w exponent of the spatial variation of flow discharge Q with upstream drainage area A, dimensionless.
Y flow depth, m.
Y_m hydraulic depth (mean flow depth), m.
y logarithm of the generic flow characteristic in the calculation of R^2.
y'' exponent of the spatial variation of GS resistance coefficient k_S with flow discharge Q, dimensionless.
y''' exponent of the spatial variation of GS resistance coefficient k_S with upstream drainage area A, dimensionless.
z''' exponent of the spatial variation of friction slope S_f with upstream drainage area A, dimensionless.
z_b channel bed elevation, m.
β channel bed inclination angle, dimensionless.
Ω cross-sectional flow area, m^2.

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